

Negative Hartman effect in one-dimensional photonic crystals with negative refractive materialsLi-Gang Wang,^{1,2} Jing-Ping Xu,^{1,3} and Shi-Yao Zhu^{1,2,3}¹*Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong*²*Department of Physics, Zhejiang University, Hangzhou, China*³*Department of Physics, Tongji University, Shanghai, China*

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The Hartman effect inside the one-dimensional photonic crystals (1DPC's) composed of negative index materials (NIM's) is always negative and is reversed to the Hartman effect inside the 1DPC's composed of positive index materials (PIM's). By calculating the phases of Fourier components of a pulse accumulated inside the 1DPC's of NIM's and the evolution of the pulse inside the 1DPC's of NIM's, the origin of the negative phase time is explained. The evolution of the electromagnetic fields inside the 1DPC's of NIM's is time reversal with conjugate to that inside the 1DPC's of PIM's for real spectral pulses. An example for the practical applications to obtain the negative phase time is illustrated.

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Photonic crystals (PC's) have attracted a lot of attention in the last decade due to their unique electromagnetic properties and potential applications [1,2]. The photonic band-gap structure (PBGs) originates from the interference of light (i.e., Bragg scattering) inside the periodic dielectric structure. Within the PBG, the electromagnetic field is evanescent. Due to the analogy between the light in the photonic band gap and an electron in a quantum barrier, the one-dimensional photonic crystals (1DPC's) acting as optical barriers were used to investigate the tunneling time [3,4]. In quantum mechanics, the tunneling time of a particle passing through a barrier is independent of the barrier length [5]. Such a phenomenon sometimes is called the Hartman effect [6], which implies superluminal and arbitrarily large (group) velocities inside long barriers. The phase time of a wave packet passing through 1DPC's composed of the positive index materials (PIM's) is always positive [3,4,7–10]. The dynamic evolution of electromagnetic fields in a 1DPC with the PIM's has been investigated in detail [11,12].

As contrasted with the PIM's, negative index materials (NIM's), which were first predicted by Veselago in 1968 [13], possess simultaneously negative permittivity ϵ and negative permeability μ . The existence of such materials with a negative refractive index ($n < 0$) was demonstrated experimentally in recent years [14], and NIM's have become a new research topic [15,16] because of their extraordinary properties such as negative refraction [17], antiparallel group, and phase velocities (backwards waves) [13]. The NIM's sometimes are also called left-handed materials (LHM's) since \vec{E} , \vec{H} , and \vec{k} form a left-handed relation. One of the most striking potential applications of the NIM's is Pendry's perfect lens [18]. Recently, the photon tunneling in 1DPC with a layer of negative refractive index was investigated [19], and it was found that the photons could tunnel through a much greater distance when a NIM is included in the 1DPC under certain conditions. In a stack of positive and negative index materials corresponding to zero (volume) averaged refractive index, a different type of PBGS, called the zero average index gap, has been found [20], which is invariant upon a change of scaling and is insensitive to the disor-

der; and an omnidirectional gap [21] is also discovered. In Ref. [34], Manga Rao and Dutta Gupta discussed the pulse propagation through the 1DPC consisting of alternating layers of PIM's and NIM's, and they showed the superluminal (or subluminal) velocities inside zero averaged index gap (or at the band-edge resonances). Theoretical calculations predicted that the photonic crystals could exhibit a negative refractive index in the near infrared [22] and optical frequencies [23].

In this paper, we consider the propagation of a light pulse passing through the 1DPC composed of NIM's. We use the transfer-matrix method [11,12,24] to calculate the transmittance and reflectance of a light pulse passing through the 1DPC's of both NIM's and the conventional PIM's. The phase time in the conventional 1DPC's (composed of PIM's) is always positive [3,4,7–10]. Contrast to the conventional Hartman effect in 1DPC's composed of PIM's, the Hartman effect in 1DPC's of NIM's is found to be negative. In order to explain this, we investigate the phase time of each Fourier component of the pulse and the dynamic evolution of the electric and magnetic fields of the pulse inside such a negative index 1DPC. We also give out an example to illustrate how to realize the negative phase time in a practical case. It should be pointed out that the effect of the negative phase time can also occur in the cases of a particle scattered by a potential well [25] and light propagation in waveguides [26].

Consider a symmetrical 1DPC with the structure of $(AB)^N A$, where A and B represent two kinds of different NIM's, and N denotes the periodic number. The relative permittivity and permeability of these two media are denoted by ϵ_i and μ_i ($i=A,B$), respectively. Please note that when $\epsilon_i < 0$ and $\mu_i < 0$, then $n_i < 0$. The thickness of each layer satisfies $|n_i|d_i = \lambda_0/4$, where λ_0 is the center wavelength of the incident pulse (corresponding to the midgap ω_0 of the 1DPC's). First, we have not considered the dispersion of NIM's, and ϵ_i and μ_i are assumed to be constant. Let a light pulse inject from vacuum into 1DPC at an incident angle θ_0 . Here we only consider the transverse electric (TE) plane-wave pulse (similar results can be obtained for transverse magnetic plane-wave pulse). For the TE wave, the electric

field \vec{E} is assumed in the x direction (dielectric layers in xy plane), and the z direction is normal to the interface of each layer. In general, the electric and magnetic fields at two positions z and $z+\Delta z$ in the same layer can be related via a transfer matrix [11,12],

$$M(\Delta z, \omega) = \begin{pmatrix} \cos[k_z \Delta z] & i \frac{\mu_i}{\eta \sqrt{\varepsilon_i \mu_i - \sin^2 \theta_0}} \sin[k_z \Delta z] \\ i \frac{\eta \sqrt{\varepsilon_i \mu_i - \sin^2 \theta_0}}{\mu_i} \sin[k_z \Delta z] & \cos[k_z \Delta z] \end{pmatrix}, \quad (1)$$

where $k_z = (\omega/c) \eta \sqrt{\varepsilon_i \mu_i - \sin^2 \theta_0}$, the sign $\eta = +1$ for PIM's and $\eta = -1$ for NIM's, c is the light speed in vacuum. Then the transmission coefficient $t(\omega)$ and reflection coefficient $r(\omega)$ can be obtained from the transfer matrix method [11,12],

$$r(\omega) = \frac{[q_0 x_{22}(\omega) - q_s x_{11}(\omega)] - [q_0 q_s x_{12}(\omega) - x_{21}(\omega)]}{[q_0 x_{22}(\omega) + q_s x_{11}(\omega)] - [q_0 q_s x_{12}(\omega) + x_{21}(\omega)]}, \quad (2)$$

$$t(\omega) = \frac{2q_0}{[q_0 x_{22}(\omega) + q_s x_{11}(\omega)] - [q_0 q_s x_{12}(\omega) + x_{21}(\omega)]}, \quad (3)$$

where $q_0 = \eta \sqrt{\varepsilon_0 \mu_0 - \sin^2 \theta_0} / \mu_0$ for the vacuum of the space $z < 0$ before the incident end and $q_s = \eta \sqrt{\varepsilon_s \mu_s - \sin^2 \theta_0} / \mu_s$ for the substrate after the exit end. Here $x_{ij}(\omega)$ ($i, j = 1, 2$) are the matrix elements of $X_N(\omega) = \prod_{j=1}^N M_j(d_j, \omega)$ which represents the total transfer matrix connecting the fields at the incident end and at the exit end. The phase time is often used to describe the pulse propagation [27]. By setting $t(\omega) = |t(\omega)| \exp[i\phi_t(\omega)]$ and $r(\omega) = |r(\omega)| \exp[i\phi_r(\omega)]$ [where the real functions $\phi_{t,r}(\omega)$ are the phases of the transmission and the reflection coefficients, respectively], the phase times for the transmitted and reflected pulses are calculated as $\tau_{t,r}(\omega) = \partial \phi_{t,r} / \partial \omega$ [28,29]. The transmitted phase shift can be obtained from $t(\omega)$ by using the method described in Ref. [30]. Let $t(\omega) = x(\omega) + iy(\omega)$, then $\phi_t = \tan^{-1}(y/x) \pm m\pi$ is the total phase accumulated as light propagating inside the medium. Here the integer m is uniquely defined by assuming that ϕ_t is a monotonic function, and the condition that $m=0$ as $\omega \rightarrow 0$ is satisfied. To calculate the phase time, we use the following way. From $t(\omega) = |t(\omega)| \exp[i\phi_t(\omega)]$, we have

$$\frac{\partial \phi_t(\omega)}{\partial \omega} = i \left(\frac{1}{|t(\omega)|} \frac{\partial |t(\omega)|}{\partial \omega} - \frac{1}{t(\omega)} \frac{\partial t(\omega)}{\partial \omega} \right). \quad (4)$$

Using $t(\omega) = \text{Re}[t(\omega)] + i \text{Im}[t(\omega)]$, we have

$$\frac{\partial |t(\omega)|}{\partial \omega} = \left[\frac{1}{|t(\omega)|} \right] \left(\text{Re}[t(\omega)] \left\{ \frac{\partial \text{Re}[t(\omega)]}{\partial \omega} \right\} + \text{Im}[t(\omega)] \times \left\{ \frac{\partial \text{Im}[t(\omega)]}{\partial \omega} \right\} \right).$$

From these relations, we finally get the transmitted phase time,

$$\tau_t(\omega) = \frac{\partial \phi_t}{\partial \omega} = \frac{1}{|t(\omega)|^2} \left(\text{Re}[t(\omega)] \frac{\partial \text{Im}[t(\omega)]}{\partial \omega} - \text{Im}[t(\omega)] \frac{\partial \text{Re}[t(\omega)]}{\partial \omega} \right). \quad (5)$$

By similar steps, we can also obtain the reflected phase time,

$$\tau_r(\omega) = \frac{\partial \phi_r}{\partial \omega} = \frac{1}{|r(\omega)|^2} \left(\text{Re}[r(\omega)] \frac{\partial \text{Im}[r(\omega)]}{\partial \omega} - \text{Im}[r(\omega)] \frac{\partial \text{Re}[r(\omega)]}{\partial \omega} \right). \quad (6)$$

From the above two equations, the transmitted and reflected phase times can be obtained from the real and imaginary parts of transmission and reflection coefficients, respectively. By Eqs. (5) and (6), we do not need to calculate the phase shifts for the transmitted and reflected light fields in order to get the phase times $\tau_{t,r}(\omega)$ at frequency ω .

In Fig. 1, we show the dependence of the transmitted phase time on the periodic number N for two kinds of 1DPC's with the symmetric structure $(HL)^N H$ and the vacuum on both sides: (a) for NIM's and (b) for PIM's at normal incidence. The transmitted phase times at the midgap of the PBG of NIM's are always negative, and tends to a *negative* constant as the periodic number N increases. This indicates that the group velocity v_g in the PBG of NIM's goes to be *negative infinity*. In the conventional 1DPC's of PIM's, however, the transmitted phase time (at ω_0) is always positive and goes to a positive constant as the periodic number N increases as shown in Fig. 1(b) [3,4]. Comparing two insets in Fig. 1, we find that there is no difference in the transmittances of these two kinds of 1DPC's because both the structures of the PBG's of NIM's and PIM's are raised from the Bragg scattering (i.e., the interference between the forward waves and the backward waves). The difference between 1DPC's of NIM's and that of PIM's is the transmitted (or reflected) phase shift accumulated during light passing

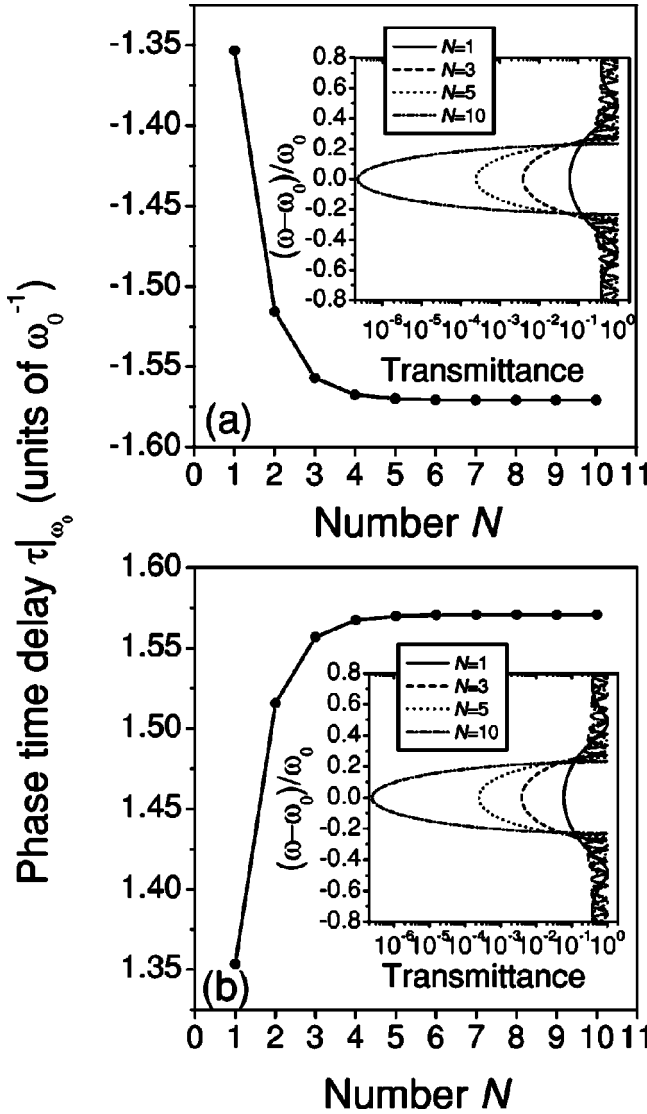


FIG. 1. The dependence of the transmitted phase time delay τ on the periodic number N at the center frequency ω_0 of the PBG with the structure $(AB)^N A$: (a) for the NIM's; (b) for the PIM's. Two insets are the transmittances for both cases, respectively. Here we choose $\epsilon_A^{NIM} = -\epsilon_B^{PIM} = -16.0$, $\mu_A^{NIM} = -\mu_B^{PIM} = -1.0$, and $\epsilon_B^{NIM} = -\epsilon_A^{PIM} = -4.0$, $\mu_B^{NIM} = -\mu_A^{PIM} = -1.0$.

through (or reflected by) the 1DPC's. For the reflected phase times, we have the results similar to that of the transmitted phase times for such lossless symmetric systems [30,31]. Therefore we may call the tunneling effect in 1DPC's with PIM's as *positive* Hartman effect, while *negative* Hartman effect in the case of NIM's.

In order to understand the negative transmitted phase time in 1DPC's of NIM's, we examine the phase shift and investigate the evolution of a light pulse propagating through a symmetric 1DPC with the structure of $(AB)^5 A$. In the case of normal incidence, for TE plane-wave light pulses, the electric field is in the x direction and the magnetic field is in the y direction. Using the method developed in Refs. [11,12], we can find the electromagnetic fields inside the 1DPC satisfying

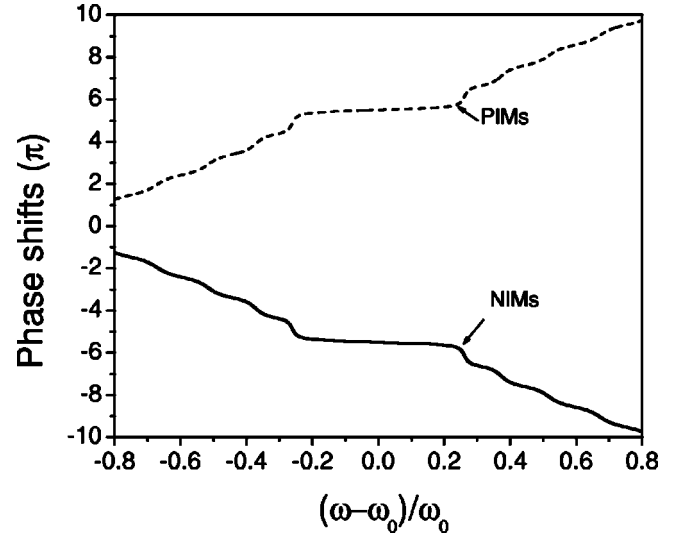


FIG. 2. The phase shifts of light passing through the 1DPC with the symmetric structure $(AB)^5 A$. Solid line for NIM's and dashed line for PIM's. Other parameters are the same as in Fig. 1.

$$E_x(z, \omega) = E^{(i)}(0, \omega) \{ [1 + r(\omega)] Q_{11}(z, \omega) + q_0 [1 - r(\omega)] Q_{12}(z, \omega) \}, \quad (7)$$

$$cH_y(z, \omega) = E^{(i)}(0, \omega) \{ [1 + r(\omega)] Q_{21}(z, \omega) + q_0 [1 - r(\omega)] Q_{22}(z, \omega) \}, \quad (8)$$

where $E^{(i)}(0, \omega)$ is the incident pulse spectrum, and $Q_{ij}(z, \omega)$ are the elements of the matrix $Q(z, \omega) = M_j(\Delta z, \omega) \prod_{i=1}^{j-1} M_i(d_i, \omega)$, where $z = \Delta z + \sum_{i=1}^{j-1} d_i$ represents any position inside the 1DPC and Δz is the distance from the point within the j th layer to the interface between j th and $(j-1)$ th layers.

For the 1DPC's composed of NIM's, when each layer satisfies the relation: $\epsilon_i^{NIM} = -\epsilon_i^{PIM}$ and $\mu_i^{NIM} = -\mu_i^{PIM}$, we have $M^{NIM}(\Delta z, \omega) = [M^{PIM}(\Delta z, \omega)]^*$, which leads to $Q_{ij}^{NIM}(z, \omega) = [Q_{ij}^{PIM}(z, \omega)]^*$, $r^{NIM}(\omega) = [r^{PIM}(\omega)]^*$ and $t^{NIM}(\omega) = [t^{PIM}(\omega)]^*$. Consequently, we have $\Lambda_\alpha^{NIM}(z, \omega) = [\Lambda_\alpha^{PIM}(z, \omega)]^*$ ($\alpha = E, H$), where $\Lambda_E(z, \omega) \equiv [1 + r(\omega)] Q_{11}(z, \omega) + q_0 [1 - r(\omega)] Q_{12}(z, \omega)$ and $\Lambda_H(z, \omega) \equiv [1 + r(\omega)] Q_{21}(z, \omega) + q_0 [1 - r(\omega)] Q_{22}(z, \omega)$. That is to say, the phase shift inside 1DPC's of NIM's is reversed to that inside 1DPC's of PIM's. In Fig. 2, we plot the phase shifts for both NIM's and PIM's cases. In the conventional 1DPC's of PIM's, the transmitted phase shift increases monotonically as frequency and is always positive. However, for the case of NIM's the transmitted phase shift is always *negative* due to the negative refractive index, and decreases monotonically as frequency increases. Therefore the transmitted phase time for NIM's is negative as shown in Fig. 1.

Now let us to look at the evolutions of the electromagnetic fields inside the 1DPC. The electric and magnetic fields in the same unit are, respectively, given by [11]

$$E_x(z, t) = \int E^{(i)}(z, \omega) e^{-i\omega t} d\omega, \quad (9)$$

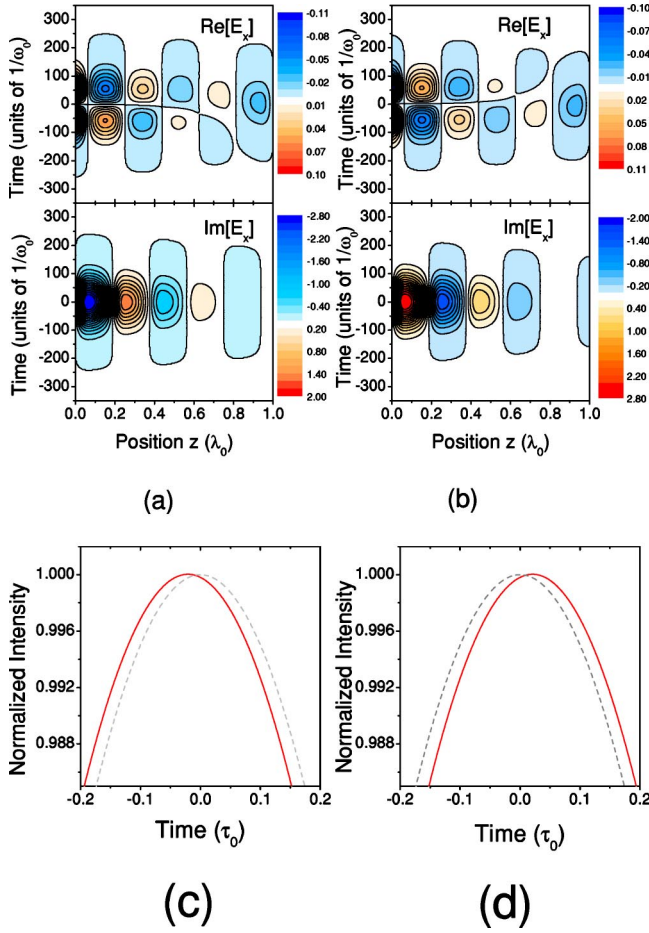


FIG. 3. (Color online) The evolution of the real and imaginary part of the electric fields inside the 1DPC of the structure $(AB)^2A$ with (a) NIM's and (b) PIM's; and the normalized intensity profiles of the incident pulse (dashed line) and the transmitted pulse (solid line) for the 1DPC's with (c) NIM's and (d) PIM's. Other parameters are the same as in Fig. 1.

$$cH_y(z,t) = \int E^{(i)}(z,\omega)e^{-i\omega t}d\omega. \quad (10)$$

From Eqs. (9) and (10), when the incident spectrum $E^{(i)}(0,\omega)$ is real, it is easy to prove the following relations: $E_x^{NIM}(t)=[E_x^{PIM}(-t)]^*$ and $H_y^{NIM}(t)=[H_y^{PIM}(-t)]^*$. It is to say that the propagation of the fields in 1DPC of NIM's is just the time reversal with conjugate comparing with the propagation of the fields inside 1DPC of PIM's. As an example, we consider the propagation of a Gaussian pulse through the 1DPC's. At the incident end, its electric field is $E_i(z=0,t)=A_0 \exp[-t^2/2\tau_0^2] \exp[-i\omega_0 t]$, where A_0 is a constant, τ_0 is the pulse width, and ω_0 is the center frequency of the pulse and is also equal to the midgap of the PBG at the normal incident case. Its Fourier spectrum is given by $E_i(z=0,\omega)=(\tau_0 A_0/2\sqrt{\pi}) \exp[-\tau_0^2(\omega-\omega_0)^2/2]$. In our numerical calculations, we take $\tau_0=56\omega_0^{-1}$ which corresponds to the spectral width $\Delta\omega \approx 0.0177\omega_0$. This means that the spectrum of the

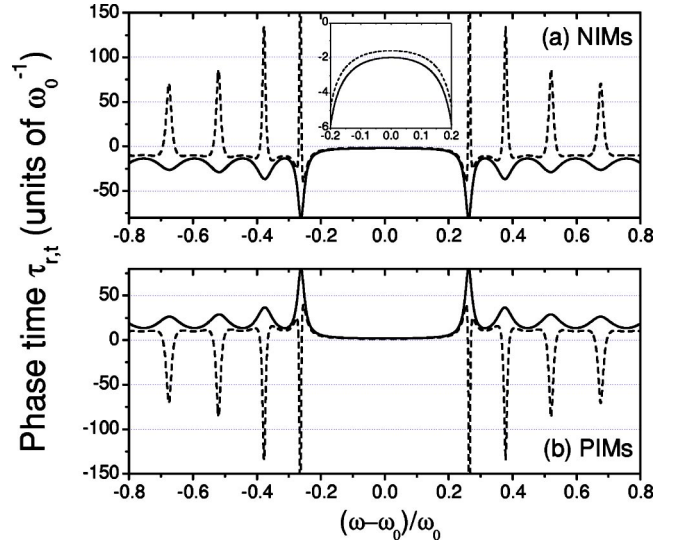


FIG. 4. The phase time delays of light injecting into the 1DPC with asymmetric structure $(AB)^2A$ -substrate: (a) for NIM's and (b) for PIM's. Solid line for $\tau_t(\omega)$ and dashed line for $\tau_r(\omega)$. Here the substrate is of $\epsilon_s=2.25$ and $\mu_s=1.0$ for both cases. Other parameters are the same as in Fig. 1.

incident pulse is very narrow, and is within the PBG's of the NIM's and PIM's 1DPC's. Figures 3(a) and 3(b) show the evolutions of the total electric fields of the light pulses inside the NIM's and PIM's 1DPC's, respectively, under the normal incident case. From Figs. 3(a) and 3(b), we find that the total electric fields in the 1DPC of NIM's are *conjugated* and *reversed* to that in the 1DPC of PIM's. There are similar results for the total magnetic field. Figure 3(c) shows the intensity profiles of the transmitted and incident pulses when the pulse propagates through the 1DPC of NIM's, and we find that the transmitted pulse is advanced before the incident pulse hitting on the interface of the incident end. Compared with the case of light pulses through the 1DPC of NIM's [as shown in Fig. 3(c)], the transmitted pulse through the 1DPC of PIM's [see Fig. 3(d)] is delayed after the incident pulse because of the positive phase time delay. Therefore the phase time for the 1DPC's of NIM's is reversed to that for the 1DPC's of PIM's. When the incident spectrum $E^{(i)}(0,\omega)$ is complex, i.e., the initial phases of the Fourier components of the incident pulse are not equal to zero, the negative phase shift induced by the 1DPC's of NIM's will reduce the initial phases, which leads to the advancement of the transmitted phase time. However, the phase shift induced by the 1DPC of PIM's always increases the initial phases of the Fourier components of the incident pulse. In a short, negative refraction index in 1DPC's leads to negative phase shifts, which makes the phase time negative, i.e., negative Hartman effect.

Due to the symmetric structure of the 1DPC that we have considered above, the reflected phase time is the same as the transmitted phase time [31]. While for the asymmetrical structures such as $(AB)^N$, $(AB)^N$ -substrate, or $(AB)^N A$ -substrate (here we assume the substrate to be different from the vacuum), the reflected phase time delay is very different from the transmitted phase time delay. For example, in Fig. 4, we plot the reflected and transmitted phase times

for the structure vacuum-(AB)⁵ A -substrate composed of (a) NIM's and (b) PIM's, respectively. Here we assume the substrate layer is semi-infinite and has the property of $\epsilon_s=2.25$ and $\mu_s=1.0$ (a nonmagnetic PIM). A light pulse enters 1DPC from the vacuum end. From Fig. 4, we know that the phase times $\tau_{t,r}(\omega)$ are different for the transmitted and the reflected fields. For the 1DPC of NIM's, in the PBG region, both of them are negative. Near the resonant peak in transmittance, we find that the reflected phase time is totally different from the transmitted phase time: for the reflected phase time, it becomes highly positive, and for the transmitted phase time, it is negative. While for the 1DPC's of PIM's, we find the result is opposite to that inside the 1DPC of NIM's: the transmitted phase time is always positive; the reflected phase time is also positive in the frequency region of the PBG, and is highly negative near the resonant transmission region. Here we would like to emphasize that $\tau_t^{NIM}(\omega)=-\tau_r^{PIM}(\omega)$ and $\tau_r^{NIM}(\omega)=-\tau_t^{PIM}(\omega)$, i.e., the time delays for NIM's are always opposite to the time delays for PIM's under the cases that each layer of the 1DPC's satisfies the relation $\epsilon_i^{NIM}=-\epsilon_i^{PIM}$ and $\mu_i^{NIM}=-\mu_i^{PIM}$. Please note that the reflected phase time $\tau_r(\omega)$ and the transmitted phase time $\tau_t(\omega)$ satisfy the relation [32] $\tau_D(\omega)=|r(\omega)|^2\tau_r(\omega)+|t(\omega)|^2\tau_t(\omega)$, where $\tau_D(\omega)$ is the dwell time at frequency ω . Obviously, for the 1DPC's of NIM's, the dwell time τ_D is negative, while τ_D is positive for that of PIM's.

Finally, we give an example to discuss the case that both ϵ_i and μ_i ($i=A$ and B) of these two materials are dispersive. Dutta Gupta *et al.* [33] investigated the transmitted phase time in the propagation of narrow-band pulses through a dispersive NIM's slab; the transmitted phase time in Ref. [33] can never be negative due to normal dispersion in the frequency range of the negative refractive index. We also find that the transmitted phase time of light pulses passing through the 1DPC of alternating layers of PIM's and NIM's (as in the cases of Refs. [20,34]) is positive due to the dispersion. In fact, from the definition of the group index $n_g(\omega)=\text{Re}[n(\omega)]+\omega d\{\text{Re}[n(\omega)]\}/d\omega$ [where $n(\omega)$ is the complex refractive index], in order to obtain the negative phase time (or group delay), there are two ways for the NIM's: the first one is to find the large negative refractive index $\text{Re}[n(\omega)]$ and the second one is to reduce the second term (that is to reduce normal dispersion $d\{\text{Re}[n(\omega)]\}/d\omega$ or even to find the value of $d\{\text{Re}[n(\omega)]\}/d\omega$ to be negative as pointed out by Mojahedi *et al.* [29]). It is reasonable to suppose that the relative permittivities and permeabilities of the two materials are $\epsilon_i=a_i-(\omega_a)_i^2/[(\omega_o)_i^2-\omega^2]$ and $\mu_i=b_i-(\omega_b)_i^2/\omega^2$ ($i=A$ and B), respectively, and the dampings of two materials are omitted. Such relative permittivities, for instance, can be realized by doping inverted two-level atoms in the GHz frequency range; and such relative permeabilities can be realized by utilizing split ring resonators [35]. In our numerical calculation, we choose $a_A=b_A=b_B=1.0$, $a_B=1.41$, $(\omega_a)_A=25$ GHz, $(\omega_a)_B=41$ GHz, $(\omega_o)_A=22$ GHz, $(\omega_o)_B=27$ GHz, $(\omega_b)_A=35$ GHz, and $(\omega_b)_B=25$ GHz. In this case, for material A , the frequency range of $\text{Re}[n_A(\omega)]<0$ is (0, 22 GHz); for material B , the range of $\text{Re}[n_B(\omega)]<0$ is (0, 27 GHz). In each period, the thicknesses of the two materials are $d_A=5$ cm and $d_B=2$ cm. Figure 5 shows the trans-

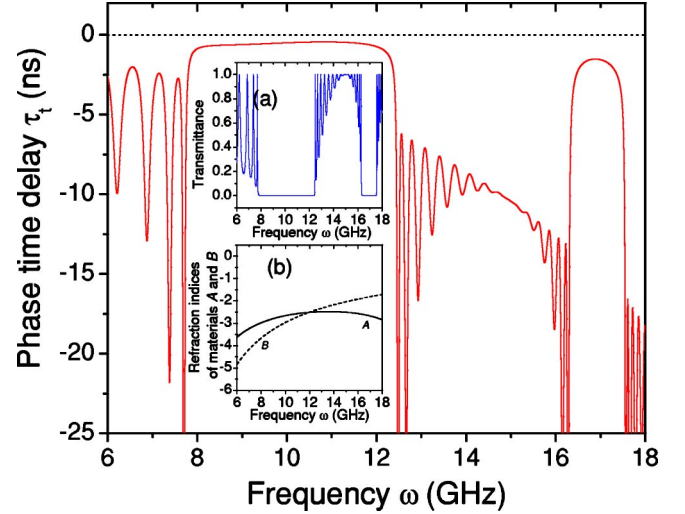


FIG. 5. (Color online) The phase time delays of light passing through the 1DPC with the structure $(AB)^{16}$. Inset (a) shows the transmittance of the 1DPC and inset (b) shows the refraction indices of materials A and B . The parameters of materials A and B are $a_A=b_A=b_B=1.0$, $a_B=1.41$, $(\omega_a)_A=25$ GHz, $(\omega_a)_B=41$ GHz, $(\omega_o)_A=22$ GHz, $(\omega_o)_B=27$ GHz, $(\omega_b)_A=35$ GHz, and $(\omega_b)_B=25$ GHz, and the thicknesses are $d_A=5$ cm and $d_B=2$ cm.

mitted phase time delay in 1DPC with the structure of $(AB)^{16}$ at the frequency range of both $\text{Re}[n_A(\omega)]<0$ and $\text{Re}[n_B(\omega)]<0$. Inset (a) shows the transmittance of such a 1DPC in the corresponding frequency range, and inset (b) shows the refraction indices for materials A and B . It is clear that the phase time delay for the transmitted light is nearly a negative constant inside the PBG's and highly negative at the edges of the PBG's. From inset (b), we see that the dispersion of the materials are not large in this example. For the practical applications, the dispersion of the NIM's should be controlled to be small in order to obtain the negative phase time delays.

In conclusion, we have investigated the Hartman effect inside the 1DPC's of NIM's. We find that the Hartman effect is always negative inside the NIM's and is reversed to the case inside the PIM's. By calculating the phase accumulated inside the 1DPC's of NIM's and the evolution of a light pulse inside the 1DPC's of NIM's, we explain why the phase times for the transmitted and reflected light pulses are negative. We have also shown that the total electromagnetic fields inside the 1DPC's of NIM's are very unusual, conjugated and reversed to that in 1DPC's of PIM's under certain conditions. For the asymmetric structure of 1DPC's with NIM's, we find that the reflected phase time is positive at the transmitted resonant region, while the transmitted phase time is always negative. An example is also given to illustrate how to obtain the negative phase time delays when the materials are dispersive.

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